(Un)knowability and Knowledge Iteration[*]

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Abstract

The κκ principle states that knowing entails knowing that one knows. The historically popular principle has fallen out of favour among many contemporary philosophers in light of putative counterexamples. Recently, some have defended more palatable versions of κκ by weakening the principle. These revisions remain faithful to their predecessor in spirit while escaping crucial objections. This paper examines the prospects of such a strategy. It is argued that revisions of the original principle can be captured by a generalized knowledge iteration principle, weak-κκ, which states that knowing entails the possibility of knowing that one knows. But weak-κκ is subject to an unknowability result and therefore must be rejected. The arguments here suggest that retreating to weaker iteration principles is not an option for the κκ enthusiast.

The κκ principle, in its original form, states that if a subject S knows some proposition p, then S knows that S knows that p. Schematically, where 'K' is the knowledge operator and 'Kp' denotes 'S knows that p', we can formulate the κκ principle as follows:

(κκ) $Kp \rightarrow KKp$

The impressive list of κκ proponents in the history of philosophy includes Plato, Aristotle, Aquinas, Spinoza, Locke, Schopenhauer and Hintikka.[1] More recently, however, a number of putative counterexamples have led many philosophers to reconsider their commitments to this iteration principle. Nevertheless some have maintained that while we should concede κκ in its original form, there are versions of the principle that are defensible. The most prominent version of the revised κκ

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[1]A more comprehensive list of κκ defenders can be found in Hintikka (1962, chapter 5).

principle states that if *S* knows that *p*, then *S* is *in a position* to know that *S* knows that *p*. Since knowing entails being in a position to know but being in a position to know does not entail knowing, this formulation is weaker than KK. Where '*Pp*' denotes '*S* is in a position to know that *p*', we can schematise this version of knowledge iteration as follows:

(PK) $Kp \rightarrow PKp$

Principles akin to PK have received endorsements to varying degrees.[2]

This paper examines the prospects of defending KK by weakening the principle. §1 motivates the knowledge iteration principle and highlights two important objections. Of primary interest is the possibility of adopting weaker forms of KK as a response to these objections. §2 proposes a generalisation of KK – WEAK-KK – which captures a wide range of possible revisions of the original principle.[3] §3 argues that WEAK-KK is incompatible with strong failures of knowledge iteration. §4 provides examples of strong KK failure. Since WEAK-KK is incompatible with strong failures of KK, the existence of strong failures of KK implies that WEAK-KK is false. §5 offers a brief discussion of where the argument here leaves the knowledge iteration enthusiast.

1 On knowing that one knows

KK has significant intuitive appeal. If we can know something while failing to know that we know it, then we are in some sense severed from our own epistemic attitudes. Consider assertions of the form '*p*, but I am unsure whether I know that *p*', and 'it is an open question whether I know that *p*, but *p*':

> If the KK principle fails – if one can know without knowing that one knows – then it's hard to see why such utterances should be infelicitous. After all, if one knows that *p*, but doesn't know that one knows that *p*, what could be wrong with dubious assertions like the ones above? How else should one express one's first-order knowledge, while acknowledging one's ignorance of whether one knows? (Greco 2015, p.667)

These Moorean-like 'dubious conjunctions' are problematic and perhaps even incoherent when asserted, but appear permissible to assert if knowledge does not iter-

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[2]See, for example, McHugh (2010, p.231), Greco (2014, pp.173-174), Stalnaker (2015, p.28), Das and Salow (2018, p.8), and Goodman and Salow (2018, p.184).

[3]WEAK-KK captures any revision of KK where knowledge is not subscripted (for example by typing or time-indexing). In general, a subscripted version of KK states that if *S* knows$_i$ that *p*, then *S* knows$_j$ (or is in a position to know$_j$) that *S* knows$_i$ that *p*, where knowing$_i$ is distinct from knowing$_j$. The plausibility of defending knowledge iteration by subscripting knowledge will mostly be set aside here, as it's not always clear whether these revisions are strictly weaker than KK. See footnote 12 and footnote 18 for discussion.

ate.[4] Furthermore, the (alleged) transparency of knowledge is sometimes thought to be closely coupled with the (alleged) transparency of belief.[5] It has even been suggested that such transparency is a requirement for epistemically responsible subjects:

> [T]ransparency… is an ideal of epistemic responsibility… if *S* is a fully idealized responsible agent and *S* knows that *p*, then *S* has duly reflected on her (first-order) epistemic states and has found it to be the case that she knows that *p*. (Cresto 2012, pp.926-927)

Arguments against κκ additionally threaten to plague common knowledge, frequently assumed across disciplines spanning linguistics, economics, game theory, and theoretical computer science.[6] Those who wish to preserve common knowledge are motivated to keep at least some version of the principle intact.

Despite these considerations in favour of knowledge iteration, κκ has been met with substantial resistance.[7] Unrestricted, κκ seems to be vulnerable to obvious counterexamples.

First, because knowledge requires belief, someone who is unreflective about what she knows may fail to have higher-order knowledge by failing to have beliefs about her first-order knowledge. A perfectly reasonable subject may know that she has hands without forming the belief that she knows this, if, for example, she does not consider whether she knows. It's dubious that even the most attentive subjects are reflective about their knowledge in such a way as to validate κκ.[8]

Second, circumstantial factors can sometimes influence one's beliefs about what one knows. Consider a modest student who is asked to produce the date that William the Conqueror landed in England on an exam. In fact, she knows that he landed in 1066. She had read this in her textbook the night before and had rehearsed this specific answer. But during the stressful exam as she's considering whether she knows, she begins to doubt herself. Questions concerning the accuracy of her memory and the reliability of her recollection give her pause. A natural judgment of this case is that while this student knows the date that William the Conqueror landed in England, she does not believe that she knows, and therefore does not know that she knows.[9]

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[4]See also Sosa (2009), Smithies (2012b), Cohen and Comesaña (2013), and Das and Salow (2018) on asserting dubious conjunctions. See Benton (2013) for a response.

[5]See Byrne (2012) and Das and Salow (2018)

[6]See, for instance, Williamson (2000, chapter 5), Stalnaker (2006), and Greco (2014).

[7]Influential arguments against the knowledge iteration principle include Alston (1980), Feldman (1981), and Williamson (2000, chapter 5).

[8]See for example Ginet (1970, pp.163-164), Feldman (1981 pp.275-276), and Nozick (1981, pp.245-246).

[9]This is a classic example from Radford (1966). The modest student may suspend judgment about whether she knows – it is not required for the student to believe that she does *not* know. Intuitions about this case can, of course, vary, and some may be inclined to challenge the orthodoxy

Attempts have been made to respond directly to both types of objections.[10] However, these immediate problems for κκ have successfully convinced many philosophers to opt for weaker forms of the principle, or to reject the principle entirely.

Those who are broadly sympathetic to these considerations against κκ but are hopeful that something in its vicinity obtains often concede the original principle and pursue slightly weaker versions. Substantive variants of κκ both circumvent these pressing challenges and manage to capture its spirit. Advocates of revision maintain that although there are instances in which knowing does not entail knowing that one knows, weaker principles, such as ΡΚ, hold universally.

2 Being in a position to know and WEAK-KK

Both the unreflective subject and the modest student (during her exam) have first-order knowledge while lacking higher-order knowledge. Plausibly, however, both are in a position to attain higher-order knowledge. All that is required for the unreflective subject to know that she knows is for her to turn her attention to her first-order knowledge. Similarly, all that is required for the modest student to know that she knows is for her to believe (on the right basis) that she knows; the student appears to lack higher-order knowledge because of the fragility of her first-order knowledge under scrutiny in special circumstances.

If *S* knows that *p*, then *S* is in a position to know that *p*. But it does not follow that *S* in fact knows that *p* whenever *S* is in a position to know that *p*. For example, although I do not currently know whether some moderately complex statement in propositional logic is a tautology, under a reasonable interpretation of 'in a position to know', I am in a position to know it – I *would* know if I were to (correctly) write out the truth tables.

The phrase 'in a position to know' is admittedly somewhat vague. Here is how some philosophers understand it:

> *S* may not believe [that *p*] or *S* may not believe [that *p*] on the basis of seeing the entailment, but then *S* will still be in a position to know [that *p*]. That is, all *S* has to do to know [that *p*] is believe it on the basis of seeing the entailment. (Cohen 1999, p.84)

> If one is in a position to know *p*, and one has done what one is in a position to do to decide whether *p* is true, then one does know *p*. The fact is open to one's view, unhidden, even if one does not yet see it. (Williamson 2000, p.95)

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that the modest student example is one in which the student has first-order knowledge while lacking higher-order knowledge. This issue will be set aside here.

[10]For example, see Hintikka (1970) and Greco (2014).

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> Being in a position to know a proposition is to be disposed to acquire the knowledge that that proposition is true, when one entertains it on the right evidential basis. (Stanley 2008, p.49)

Whatever the sufficient conditions are, it seems that a necessary condition for one to be in a position to know is that it's *possible* for one to know.[11] To deny this is to allow for situations in which one is in a position to know something that is impossible for one to know. It's difficult to see how '*S* is in a position to know that *p*, but it is in fact impossible for *S* to know that *p*' can be faithfully maintained. More formally, where '$\Diamond Kp$' denotes 'it is possible for *S* to know that *p*', we have:

(POSITION) $Pp \rightarrow \Diamond Kp$

PK and POSITION entail WEAK-KK:

(WEAK-KK) $Kp \rightarrow \Diamond KKp$

WEAK-KK states that if *S* knows that *p*, then it's possible for *S* to know that *S* knows that *p*; it is a generalised version of KK.[12] WEAK-KK is implied by (but does not imply) PK and can readily accommodate further modifications. Consider:

> … if *S* knows *p*, and *S* grasps the proposition that she knows *p*, and the normal conditions for psychological self-knowledge are in place, then *S* is in a position to know that she knows *p*… (McHugh 2010, p.231)

> For any agent who is able to apply KNOW [the rule 'if *p*, believe that you know that *p*'] to the premise that *p*, if the agent knows that *p*, she is in a position to know that she knows that *p*. (Das and Salow 2018, p.8)

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[11]Spencer (2017, pp.489-491) dissents. The notion of possibility employed here can be understood as metaphysical (or even logical) possibility. It is therefore possible for *S* to know that *p* even if *S* is physically or psychologically incapable of knowing that *p*. This allows POSITION to capture the different uses of 'in a position to know', for example, in cases where the subjects in question are in a position to know something that they, in some sense, cannot know because of certain contingent limitations, as discussed by Gibbons (2006, pp.28-29) and Smithies (2012a, p.733).

[12]As it was mentioned in footnote 3, WEAK-KK is not *fully* general, as it does not capture revisions of KK which rely on subscripting knowledge. Strategies for subscripting knowledge include distinguishing between perceptual and reflective knowledge, adopting a hierarchical approach in which one can know$_i$ that *p* without knowing$_j$ that *p* where *i* and *j* refer to different levels in the hierarchy and fragmenting knowledge on the basis of when it is available for certain practical purposes. See for instance Sharon and Spectre (2008), Dokic and Egré (2009), Cresto (2012), and Greco (2015). Alternatively, as two referees suggested, one might be sympathetic to a time-indexed version of KK according to which when one knows at time *i* that *p*, then one is in a position to know at some later time *j* that one knew at *i* that *p*. Although we are assuming that knowledge is not subscripted in these ways, certain revisions which rely on subscripting will remain susceptible to the arguments here. See footnote 18.

These formulations stipulate different qualifications to ᴘᴋ, but agree that knowing implies being in a position to know that one knows when some set of conditions obtains. It is implicit that these conditions (whatever they are) *can* be met. Therefore these, and formulations like these, will imply ᴡᴇᴀᴋ-ᴋᴋ.

The generality of ᴡᴇᴀᴋ-ᴋᴋ encourages creativity from the ᴋᴋ defender by allowing the possibility operator to capture any additional conditions, requirements, or qualifications. Subjects can be idealised beyond the scope of what is usually intended by 'in a position to know'. Not only would ᴡᴇᴀᴋ-ᴋᴋ resolve complications involving subjects who are unreflective about their first-order knowledge, and fail to attain higher-order knowledge in peculiar contexts, but it would also be a panacea for other types of objections against ᴋᴋ. ᴡᴇᴀᴋ-ᴋᴋ permits one to be informed by an oracle that one has higher-order knowledge whenever one has first-order knowledge.[13] But generality is a double-edged sword, for if ᴡᴇᴀᴋ-ᴋᴋ is untenable, then any revision of ᴋᴋ captured by ᴡᴇᴀᴋ-ᴋᴋ will also be untenable.[14]

3   Unknowability

It is provable that, given several plausible assumptions, if all truths are knowable, then all truths are in fact known.[15] Since evidently not all truths are in fact known, not all truths are knowable. There are propositions which are true but unknowable. While some have found this conclusion objectionable and take issue with the different premisses employed in the argument, others (especially those who reject verificationism) have welcomed the result.[16]

Unknowability also infects ᴡᴇᴀᴋ-ᴋᴋ. It can be shown that ᴡᴇᴀᴋ-ᴋᴋ is inconsistent with *strong* ᴋᴋ failure. S is in a state of strong ᴋᴋ failure just in case S knows that p, but also knows that S does not know that S knows that p (that is, $Kp \land K\neg KKp$). This Fitch-like result requires four assumptions. First, that knowledge is factive:

(ꜰᴀᴄᴛɪᴠɪᴛʏ) $\vdash Kp \rightarrow p$

Second, a weak principle of distribution for knowledge:

(ᴅɪsᴛʀɪʙᴜᴛɪᴏɴ) $\vdash K^n(p \land q) \rightarrow \Diamond(K^n p \land K^n q)$

Third, a weak principle of agglomeration for knowledge:

(ᴀɢɢʟᴏᴍᴇʀᴀᴛɪᴏɴ) $\vdash (K^n p \land K^n q) \rightarrow \Diamond K^n(p \land q)$

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[13]ᴡᴇᴀᴋ-ᴋᴋ therefore straightforwardly addresses complications for ᴋᴋ that involve reliable subjects who do not know about their own reliability, and margin-for-error principles.

[14]ᴡᴇᴀᴋ-ᴋᴋ also accounts for principles that directly modify ᴋᴋ as opposed to ᴘᴋ as defended by, for example, Prichard (1950, p.86), Ginet (1970, p.163), and Chisholm (1989, p.100).

[15]See Fitch (1963, p.138).

[16]For discussion, see the collection of papers in Salerno (2009).

Fourth, the following inference rule:

(NECESSITATION) From $\vdash p$ infer $\vdash \Box p$.

FACTIVITY is uncontroversial, for if S knows that p, then p is true. DISTRIBUTION states that if S has n iterations of knowledge of the conjunction p and q, then it is *possible* for S to have n iterations of knowledge of each conjunct. AGGLOMERATION states that if S has n iterations of knowledge of p and q individually, then it is *possible* for S to have n iterations of knowledge of the conjunction. NECESSITATION allows us to infer that $\Box p$ is a theorem whenever p is a theorem. We will also make use of the following fact:

(DUALITY) $\vdash \Box p \leftrightarrow \neg \Diamond \neg p$

DUALITY is standard in all (classical) modal logics.[17]

The following establishes that WEAK-KK is inconsistent with strong KK failure. What will be shown is that if WEAK-KK is true, then strong KK failures are impossible. But – as subsequent examples will demonstrate – strong KK failures *are* possible, so WEAK-KK is false.

Because $KKp \wedge \neg KKp$ is a contradiction, its negation is a theorem:

(1) $\vdash \neg(KKp \wedge \neg KKp)$

But notice by two applications of FACTIVITY:

(2) $KKp \wedge KK\neg KKp \vdash KKp \wedge \neg KKp$

Therefore by propositional logic, (1) and (2):

(3) $\vdash \neg(KKp \wedge KK\neg KKp)$

By NECESSITATION and (3):

(4) $\vdash \Box\neg(KKp \wedge KK\neg KKp)$

By DUALITY, (4) is equivalent to:

(5) $\vdash \neg\Diamond(KKp \wedge KK\neg KKp)$

The contrapositive of DISTRIBUTION and (5) entail:

(6) $\vdash \neg KK(p \wedge \neg KKp)$

By NECESSITATION and (6):

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[17]See Chellas (1980, chapter 4) for an overview of the formal details. It's worth noting just how weak the assumptions of DISTRIBUTION and AGGLOMERATION are; often, $K^n(p \wedge q)$ is taken to be equivalent to $K^n p \wedge K^n q$.

$$(7) \vdash \Box \neg KK(p \land \neg KKp)$$

By DUALITY, (7) is equivalent to:

$$(8) \vdash \neg \Diamond KK(p \land \neg KKp)$$

Assuming WEAK-KK, the contrapositive of WEAK-KK and (8) entail:

$$(9) \vdash \neg K(p \land \neg KKp)$$

By NECESSITATION and (9):

$$(10) \vdash \Box \neg K(p \land \neg KKp)$$

By DUALITY, (10) is equivalent to:

$$(11) \vdash \neg \Diamond K(p \land \neg KKp)$$

The contrapositive of AGGLOMERATION and (11) entail:

$$(12) \vdash \neg(Kp \land K\neg KKp)$$

Given the assumptions of FACTIVITY, DISTRIBUTION, AGGLOMERATION, NECESSITATION, and WEAK-KK, it follows that strong KK failures are impossible. There are very compelling reasons to accept FACTIVITY, DISTRIBUTION, AGGLOMERATION, and NECESSITATION. Therefore if there are cases of strong KK failure, then WEAK-KK must be rejected. The Appendix extends the result here to show that WEAK-KK is inconsistent with *anything* of the form $Kp \land K\neg K^n p$.

4   Strong failures of KK

Knowledge iteration failures exhibit instances of $Kp \land \neg KKp$. Those who retreat to some version of the iteration principle which entails WEAK-KK must be content with instances of KK failure – this is a consequence of conceding the original principle. But those who accept instances of $Kp \land \neg KKp$ because of counterexamples like the unreflective subject or the unconfident student are pressured to accept strong failures of KK when those counterexamples are extended. If there are examples of $Kp \land K\neg KKp$ (or $Kp \land K\neg K^n p$ more generally), then WEAK-KK must be abandoned.

4.1   (Un)reflective Lee

The unreflective subject who never considers her first-order knowledge fails to have beliefs about what she knows, and therefore lacks higher-order knowledge. But even the most attentive subject may fail to believe that she knows that she knows that she knows… above some level. Consider a subject who is aware of this fact.

Lee knows many ordinary propositions. She knows that h, she has hands. She is exceptionally reflective about her knowledge. She not only knows that h, but she also believes that she knows, and perhaps knows that she knows, and believes that she knows that she knows. But Lee also knows that irrespective of how attentive or reflective she is, she has no n-level beliefs for large (finite) values of n. She knows, for example, that she does not have millionth-level beliefs about her knowledge, and reasons from this to the conclusion that she does not have millionth-order knowledge that h. Lee is in a state of $Kh \wedge K\neg K^n h$ for some n.

4.2 Unconfident Velma

The unconfident student knows the answer to a certain question on her exam, but in peculiar circumstances, is reluctant to believe that she knows because of her natural dispositions. Consider an unconfident but reflective student who is in such a situation.

Suppose that Velma is taking an exam which asks her for the date when William the Conqueror landed in England. As it so happens, Velma learned that William the Conqueror landed in England in 1066 from a book she recently read. Although her memory of the date is reliable and formed on the right basis, Velma is of a modest and nervous disposition; during the exam, the enormous pressure she feels affects her in such a way that she refrains from believing that she knows the answer. While the date 1066 comes to her mind, Velma suspects that this is actually a guess out of desperation. She does not remember reading it in her book. When Velma reflects on her situation, she recognises that she cannot determine whether she knows the date or whether she is merely guessing.

Velma knows that w, William the Conqueror landed in England in 1066. Under ordinary circumstances, it may be granted that she has both first-order knowledge and higher-order knowledge. But she fails to know that she knows that w when she is asked to produce the answer on her exam, for she does not believe that she knows that w. Upon reflection, Velma comes to recognise that she does not know that she knows that w. Velma is in a state of $Kw \wedge K\neg KKw$.

5 From kk to weak-kk and back

weak-kk does little by way of offering refuge for kk defenders. Those who are moved by putative counterexamples to kk cannot hope to salvage the principle by endorsing a weakened revision of knowledge iteration captured by weak-kk. This is because weak-kk is exposed to an (un)knowability 'paradox'. This Fitch-like result depends on there being instances of strong kk failure. As it was argued, the counterexamples to kk can be extended to generate instances of strong kk failure.

Defenders of weak-kk (or an iteration principle captured by weak-kk) must reject

one of the assumptions required for the unknowability result, or reject the possibility of strong ᴋᴋ failure. Denying the former incurs serious costs.[18] Denying the latter requires principled reasons for why strong ᴋᴋ failures cannot arise. The battle for ᴋᴋ is an uphill one, but as the arguments here suggest, retreat is not an option.

Appendix: Generalised unknowability

In the main text, we proved that ᴡᴇᴀᴋ-ᴋᴋ is inconsistent with $Kp \wedge K\neg K^2 p$. Assuming ᴡᴇᴀᴋ-ᴋᴋ, we have $\neg(Kp \wedge K\neg K^2 p)$ as a theorem. In the appendix, we will prove the general result that if $\neg(Kp \wedge K\neg K^i p)$ is a theorem, then $\neg(Kp \wedge K\neg K^{i+1} p)$ is a theorem. As a consequence, ᴡᴇᴀᴋ-ᴋᴋ is inconsistent with $Kp \wedge K\neg K^n p$ for any $n > 1$. We will make convenient use of the fact that, given ɴᴇᴄᴇssɪᴛᴀᴛɪᴏɴ, ᴅᴜᴀʟɪᴛʏ and propositional logic, if $\neg p$ is a theorem, then $\neg\Diamond p$ is a theorem. The proof proceeds by induction.

Suppose that $\neg(Kp \wedge K\neg K^i p)$ is a theorem:

(1) $\vdash \neg(Kp \wedge K\neg K^i p)$

When we uniformly substitute Kp for p in (1), we have:

(2) $\vdash \neg(KKp \wedge K\neg K^{i+1} p)$

Notice that by ғᴀᴄᴛɪᴠɪᴛʏ:

(3) $KKp \wedge KK\neg K^{i+1} p \vdash KKp \wedge K\neg K^{i+1} p$

Therefore by propositional logic, (2) and (3):

(4) $\vdash \neg(KKp \wedge KK\neg K^{i+1} p)$

By ɴᴇᴄᴇssɪᴛᴀᴛɪᴏɴ, ᴅᴜᴀʟɪᴛʏ and (4):

(5) $\vdash \neg\Diamond(KKp \wedge KK\neg K^{i+1} p)$

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[18]It might be suggested that these Fitch-like results motivate subscripting knowledge in some way. See for example Paseau (2008), Linsky (2009), and Cresto (2017); see Williamson (2000, chapter 12) and Halbach (2008) for criticism. Strictly typing knowledge prevents the original derivation of Fitch's 'paradox' and, as a consequence, also the results here. Notice, however, that on any account of subscripting knowledge in which propositions of the form $K_i p \wedge K_i \neg K_j K_i p$ are coherent, it is provable that these propositions are unknowable$_j$. At least on some relevant proposals of subscripting, we can generate cases of $K_i p \wedge K_i \neg K_j K_i p$ by modifying examples like (un)reflective Lee and unconfident Velma. For instance, consider a time-indexed version of knowledge iteration. Suppose at time i Lee knows both that h and that she will never have beliefs about her knowledge that h for any future time j. Lee would therefore be in a state of $K_i h \wedge K_i \neg K_j K_i h$. Or, suppose at time i Velma knows that w and also that at future time j when she's taking her exam, she will be unsure whether she knows that w because of her dispositions. Velma would then be in a state of $K_i w \wedge K_i \neg K_j K_i w$. Similar examples can be produced to address various other kinds of subscripting.

The contrapositive of DISTRIBUTION and (5) entail:

$$(6) \vdash \neg K K(p \wedge \neg K^{i+1} p)$$

By NECESSITATION, DUALITY and (6):

$$(7) \vdash \neg \Diamond K K(p \wedge \neg K^{i+1} p)$$

The contrapositive of WEAK-KK and (7) entail:

$$(8) \vdash \neg K(p \wedge \neg K^{i+1} p)$$

By NECESSITATION, DUALITY and (8):

$$(9) \vdash \neg \Diamond K(p \wedge \neg K^{i+1} p)$$

The contrapositive of AGGLOMERATION and (9) entail:

$$(10) \vdash \neg(K p \wedge K \neg K^{i+1} p)$$

This completes the proof, for we have shown that if $\neg(K p \wedge K \neg K^i p)$ is a theorem, then $\neg(K p \wedge K \neg K^{i+1} p)$ is a theorem, as desired.

WEAK-KK is incompatible with $K p \wedge K \neg K^n p$ for any $n > 1$ (and WEAK-KK is also trivially incompatible with $K p \wedge K \neg K^n p$ when $n = 1$ because $K p \wedge K \neg K p$ entails the contradiction $K p \wedge \neg K p$). If it is possible for S to know that p while also knowing that S does not know that S knows that S knows that… S knows that p, then WEAK-KK is false.

References

[1] Alston, W. P. 1980. Level-Confusions in Epistemology. *Midwest Studies in Philosophy*, 5, pp.135-150.

[2] Benton, M. A. 2013. Dubious Objections for Iterated Conjunctions. *Philosophical Studies*, 162, pp.355-358.

[3] Byrne, A. 2012. Knowing What I See. In *Introspection and Consciousness*, eds. D. Smithies and D. Stoljar, pp.183-210. Oxford: Oxford University Press.

[4] Chellas, B. F. 1980. *Modal Logic: An Introduction*. Cambridge: Cambridge University Press.

[5] Chisholm, R. M. 1989. *Theory of Knowledge*, 3rd edition. Englewood Cliffs, New Jersey: Prentice-Hall.

[6] Cohen, S. 1999. Contextualism, Skepticism, and the Structure of Reasons. *Philosophical Perspectives*, 13, pp.57-89.

[7] Cohen, S. and J. Comesaña. 2013. Williamson on Gettier Cases and Epistemic Logic. *Inquiry*, 56, pp.15-29.

[8] Cresto, E. 2012. A Defense of Temperate Epistemic Transparency. *Journal of Philosophical Logic*, 41, pp.923-955.

[9] Cresto, E. 2017. Lost in Translation: Unknowable Propositions in Probabilistic Frameworks. *Synthese*, 194, pp.3955-3977.

[10] Das, N. and B. Salow. 2018. Transparency and the KK Principle. *Noûs*, 52, pp.3-23.

[11] Dokic, J. and P. Égré. 2009. Margin for Error and the Transparency of Knowledge. *Synthese*, 166, pp.1–20.

[12] Feldman, R. 1981. Fallibilism and Knowing that One Knows. *Philosophical Review*, 90, pp.266-282.

[13] Fitch, F. 1963. A Logical Analysis of Some Value Concepts. *The Journal of Symbolic Logic*, 28, pp.135-142.

[14] Gibbons, J. 2006. Access Externalism. *Mind*, 115, pp.19-39.

[15] Ginet, C. 1970. What Must be Added to Knowing to Obtain Knowing that One Knows? *Synthese*, 21, pp.163-86.

[16] Goodman, J. and B. Salow. 2018. Taking a Chance on KK. *Philosophical Studies*, 175, pp.183-196.

[17] Greco, D. 2014. Could KK be OK? *Journal of Philosophy*, 111, pp.169-197.

[18] Greco, D. 2015. Iteration and Fragmentation. *Philosophy and Phenomenological Research*, 91, pp.656-673.

[19] Halbach, V. 2008. On a Side Effect of Solving Fitch's Paradox by Typing Knowledge. *Analysis*, 68, pp.114-120.

[20] Hintikka, J. 1962. *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Ithaca: Cornell University Press.

[21] Hintikka, J. 1970. 'Knowing that one Knows' Reviewed. *Synthese*, 21, pp.141-162.

[22] linsky, B. 2009. Logical Types in Some Arguments about Knowability and Belief. In *New Essays on the Knowability Paradox*, ed. J. Salerno, pp.163-179. Oxford: Oxford University Press.

[23] McHugh, C. 2010. Self-knowledge and the KK Principle. *Synthese*, 173, pp.231–257.

[24] Nozick, R. 1981. *Philosophical Explanations*. Cambridge: Harvard University Press.

[25] Paseau, A. 2008. Fitch's Argument and Typing Knowledge. *Notre Dame Journal of Formal Logic*, 49, pp.153-176.

[26] Prichard, H. A. 1950. *Knowledge and Perception: Essays and Lectures*. Oxford: Clarendon Press.

[27] Radford, C. 1966. Knowledge – By Examples. *Analysis*, 27, pp.1-11.

[28] Salerno, J., ed. 2009. *New Essays on the Knowability Paradox*. Oxford: Oxford University Press.

[29] Sharon, A. and L. Spectre. 2008. Mr. Magoo's Mistake. *Philosophical Studies*, 139, pp.289-306.

[30] Smithies, D. 2012a. Mentalism and Epistemic Transparency. *Australasian Journal of Philosophy*, 90, pp.723-741.

[31] Smithies, D. 2012b. Moore's Paradox and the Accessibility of Justification. *Philosophy and Phenomenological Research*, 85, pp.273-300.

[32] Sosa, D. 2009. Dubious Assertions. *Philosophical Studies*, 146, pp.269-72.

[33] Spencer, J. 2017. Able to do the Impossible. *Mind*, 126, pp.466-497.

[34] Stalnaker, R. 2006. On Logics of Knowledge and Belief. *Philosophical Studies*, 128, pp.169-199.

[35] Stalnaker, R. 2015. Luminosity and the KK Thesis. In *Externalism, Self-Knowledge, and Skepticism*, ed. S. C. Goldberg, pp.19-40. Cambridge: Cambridge University Press.

[36] Stanley, J. 2008. Knowledge and Certainty. *Philosophical Issues*, 18, pp.35-57.

[37] Williamson, T. 2000. *Knowledge and Its Limits*. Oxford: Oxford University Press.